

Simple Harmonic Motion

Object: To determine the force constant of a spring and then study the harmonic motion of that spring when it is loaded with a mass m .

Apparatus: Force sensor, motion sensor, spring, weight hanger and weights, meter stick, support hardware, interface device, computer, and *DataStudio* Software.

Foreword

From lecture you have determined that the restoring force exerted by a spring is

$$(1) \quad F_{\text{spring}} = -ky$$

where y is the displacement from equilibrium and k is the spring constant.

Recall that the equation of motion of an object is given by

$$(2) \quad F = ma$$

If the force acting on the mass m is the restoring force of the spring, then combining equations (1) and (2), we obtain

$$(3) \quad ma = -ky$$

If you are in a calculus-based class, you may prefer to write equation (3) as follows:

$$(4) \quad (d^2y/dt^2) + (k/m)y = 0$$

In class, you will determine that the solution to equation (3) and/or (4) is

$$(5) \quad y = A \sin \omega t$$

if

$$(6) \quad \omega = (k/m)^{1/2} \quad \text{or} \quad T = 2\pi(m/k)^{1/2}$$

where ω is the angular frequency and T is the period.

You will also establish that

$$(7) \quad v = \omega A \cos \omega t = v_{\text{max}} \cos \omega t$$

and

$$(8) \quad a = -\omega^2 A \sin \omega t = -a_{\text{max}} \sin \omega t$$

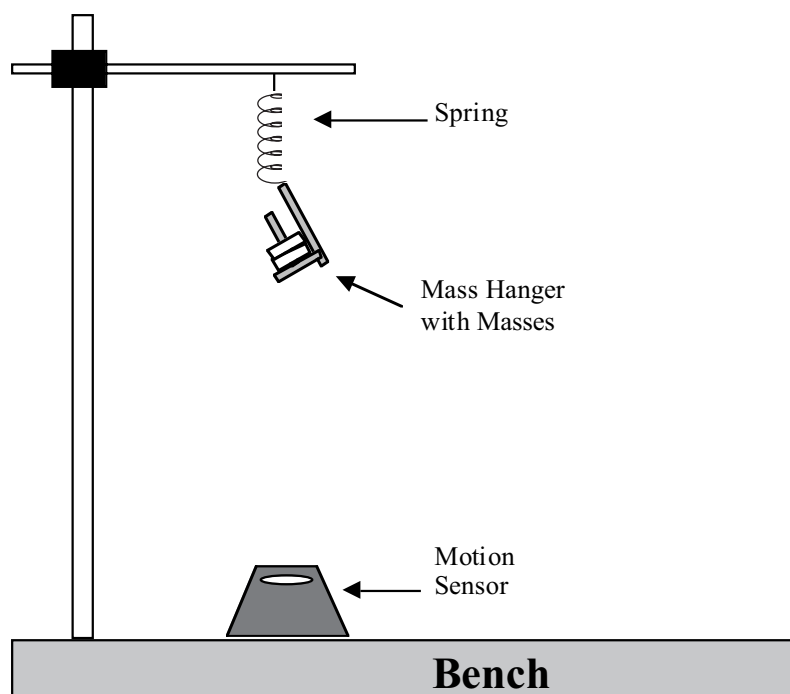
In this experiment you will:

- determine k , the spring constant.
- determine that $y = A \sin \omega t$ is indeed a solution to equation (3) and/or (4) by showing that $T = 2\pi(m/k)^{1/2}$.
- determine v_{max} and a_{max} both experimentally and theoretically.

Procedure



Part I: Hardware Set-up

1. Plug the yellow lead of the **Motion Sensor** into a digital port of the interface, and plug the black lead into the adjacent digital port.
2. Hang the spring over the table from the provided supports. Position the horizontal support near the top of the vertical support. Throughout the experiment, the mass hanger must come no closer than a distance of 20 cm from the motion sensor to obtain accurate data. In future parts of the experiment, the mass hanger will be oscillating above the motion detector.
3. Hang the mass hanger from the spring.
4. Position the **Motion Sensor** on the bench directly under the mass hanger.





Part II: Software Setup

1. Open **DataStudio**. Click Create Experiment.
2. Tell the software that a **Motion Sensor** is plugged into the interface box by dragging the motion sensor icon to the correct digital channel.
3. Since we intend to use the **Motion Sensor** as a digital meter stick, we need a digital meter to tell us the distance from the motion sensor to the weight hanger on the spring. We can accomplish this by dragging the **Digits** icon onto the **motion sensor** icon.
4. Go to the Experiment option at the top of the window and select monitor data. Now your digital meter stick is working.
5. Record the distance from the motion sensor to the mass hanger as x_0 . Make sure that the **Motion Sensor** is 'seeing' the bottom of the mass hanger.

6. Now place a mass $m_1 = 0.0200$ kg on the weight hanger and record the new distance to the weight hanger as x_1 . **Do not include the mass of the weight hanger in m_1 ; the mass of the weight hanger is incorporated in the equilibrium point x_0 .** (Note: If 0.0200 kg is not sufficient to change the displacement as compared with the weight hanger alone, start with 0.040 kg). When a mass of $m_1 = 0.0200$ kg is placed on the spring, the spring is stretched an amount $\Delta x_1 = x_0 - x_1$, and the force causing this amount of stretch is $F_1 = m_1 g$. Now place a mass $m_2 = 0.0400$ kg on the weight hanger and record the new distance to the weight hanger as x_2 . When a mass of $m_2 = 0.0400$ kg is placed on the spring, the spring is stretched an amount $\Delta x_2 = x_0 - x_2$, and the force causing this stretch is $F_2 = m_2 g$. Continue adding mass to the weight hanger in 0.0200 kg increments until you have a total mass of 0.1000 kg on the weight hanger. For each mass, record the new position and calculate the total stretch and the total force which caused this stretch in Table 1.
7. Stop monitoring the distance from the motion sensor to the weight hanger.
8. Create a graph of your data from Table 1.
 - a. First click on the *Experiment* pull down menu located near the top of the *DataStudio* window.
 - b. Select the *New Empty Data table ...*
 - c. You should now have a **Data Table** with an X column and a Y column.
 - d. Enter the Stretch of the Spring from your data table in the X column and the corresponding Force of Gravity in the Y column.
 - e. Drag the graph icon to the icon labeled **Editable Data**.
 - f. From the Fit menu, select Linear. Convince yourself that the slope of this graph is the spring constant k .
 - g. Print the graph.
 - h. Manually label each axis on your printed graph in order to clearly identify each axis beyond the computer generated labels of X and Y. Manually provide a title for your graph.
9. Now that we have determined the spring constant, we are ready to obtain plots of position, velocity, and acceleration vs. time for the oscillating mass.
10. Close the **Digits** Window.
11. Drag the graph icon to the position icon that appears under the Data window. Add velocity and acceleration to the graph.
 - a. Make sure the x-axis locked button  is selected.
12. Hang a 0.085 kg mass on the spring (Note: Do not forget about the 0.005 kg mass of the weight hanger; we must now account for the total mass that will be oscillating.). Start the mass oscillating by pulling it down (from its new equilibrium position) by 0.10 to 0.15 m and then releasing. Click the Start button. Collect data for 5 to 10 seconds, and then hit the Stop button.
13. On the graph window, select the statistics pull down menu. Select **maximum** and **minimum** and **apply to all**.
14. Use the smart tool  to determine the period, T of the oscillation.
15. Print your graphs.
16. Complete Tables 2 and 3.
17. Verify that $v = \frac{dy}{dt} = A\omega \cos \omega t$ and $a = \frac{d^2 y}{dt^2} = \frac{dv}{dt} = -A^2 \omega \sin \omega t$. (Hint: transform the position graph into a sine graph by positioning the origin of a y and t coordinate system appropriately. Once you have transformed the position graph into a sine graph, extend the y-axis that you drew down through the velocity and acceleration graphs. Using the existing t-axis of the velocity and acceleration graphs and your y-axis, identify the trigonometric function that is formed within each graph.) How does this compare to the equations in the Forward on page 9M-1?

Part III: Verifying that the Period (and Frequency) is Independent of the Initial Displacement

1. Before proceeding with the next part of the lab, make sure you have printed your graph containing the plots of position, velocity, and acceleration.
2. Select the *Experiment* pull down menu located near the top of the *DataStudio* window.
3. Select *Delete All Data Runs*, and delete your data.
4. Delete the graph containing the plots of position, velocity, and acceleration.
5. Create a new graph of velocity by dragging the graph icon to the velocity icon that appears under the Data window.
6. Pull the 0.085 kg mass down approximately 10cm, and without pressing the *Start* button observe the motion of the mass.
7. Stop the mass.
8. Now, pull the mass down approximately 15cm, and again observe the motion of the mass without pressing the *Start* button.
9. Did the two different displacements cause the mass to oscillate at different frequencies? If yes, which displacement do you believe caused the mass to oscillate at the greater frequency? Record your opinion on your data sheet.
10. Pull the mass down approximately 10cm and let the mass begin to oscillate. Click the *Start* button when the mass is as near the bottom of the oscillation as possible.
11. After you have recorded approximately 4 peaks, click the *Stop* button.
12. Now pull the mass down approximately 15 cm and let the mass begin to oscillate.
13. Again click the *Start* button when the mass is as near the bottom of the oscillation as possible, and record approximately 4 peaks before pressing the *Stop* button.
14. Adjust the scale of each axis using the *autoscale button* .
15. Using the smart tool , determine the Period of each plot.
16. Record the periods for each displacement on the Data Sheet in Table 4.

DATA AND CALCULATION SUMMARY**Table 1**

Mass added to weight hanger	Distance from sensor to mass	Stretch of Spring	Force of gravity = Force of spring = mg
	$x_0 =$		
$m_1 = 0.020 \text{ kg}$	$x_1 =$	$\Delta x_1 = x_0 - x_1 =$	$F_1 = m_1 g =$
$m_2 = 0.040 \text{ kg}$	$x_2 =$	$\Delta x_2 = x_0 - x_2 =$	$F_2 = m_2 g =$
$m_3 = 0.060 \text{ kg}$	$x_3 =$	$\Delta x_3 = x_0 - x_3 =$	$F_3 = m_3 g =$
$m_4 = 0.080 \text{ kg}$	$x_4 =$	$\Delta x_4 = x_0 - x_4 =$	$F_4 = m_4 g =$
$m_5 = 0.100 \text{ kg}$	$x_5 =$	$\Delta x_5 = x_0 - x_5 =$	$F_5 = m_5 g =$

The value for $k =$

Table 2 (From the Plots)

$y_{\max} =$	
$y_{\min} =$	
$T =$	
$v_{\max} =$	
$a_{\max} =$	

Table 3 (Calculations)

Amplitude of oscillation $A = (y_{\max} - y_{\min})/2 =$	
Period = $T = 2\pi(m/k)^{1/2} =$	
Linear frequency $f = 1/T =$	
Angular frequency $\omega = 2\pi f =$	
$v_{\max} = \omega A =$	
$a_{\max} = \omega^2 A =$	
Compare v_{\max} calculated with v_{\max} from the plot	
Compare a_{\max} calculated with a_{\max} from the plot	

What trig function is formed on the velocity graph?

What trig function is formed on the acceleration graph?

How do these trig functions compare to the equations in the Forward on page 9M-1?

Examine the position versus time plot and comment on the value of the velocity and acceleration when the mass is in the following four positions: (Use general descriptors such as maximum, minimum, zero, increasing, decreasing, etc.)

The mass on the spring is at its maximum positive position.

The mass on the spring is at the equilibrium position and moving downward.

DATA AND CALCULATION SUMMARY

The mass on the spring is at its maximum negative position.

The mass on the spring is at the equilibrium position and traveling upward.

Did the two different displacements cause the mass to oscillate at different frequencies? If yes, which displacement do you believe caused the mass to oscillate at the greater frequency?

Table 4

Displacement	Period (s)	Frequency (1/s or Hz)
Approximately 10 cm		
Approximately 15 cm		

How do the results from Table 4 compare to you initial opinion?

Justify the results from Table 4.

